

# Inverse Problems and Image Reconstruction

ECTS: 6

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UNIVERSITY WHERE THE COORDINATOR IS: UC3M

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? No

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UNIVERSITY WHERE THE LECTURER 1 IS: UPM

HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES? No

# SUBJECT CONTENTS

# Introduction and Basic Notions

- Direct and inverse problems
- Well and ill-posed problems
- Existence and uniqueness of the solution
- Stability

#### Least squares

- Motivation and general idea
- Applications

### Regularization

- Motivation and general idea
- Tikhonov, Lardy and Landweber algorithms
- Morozov's discrepancy principle

#### Singular Value Decomposition

- Theoretical background, meaning and properties
- Noise filtering and data reconstruction
- Linear systems and regularization
- Extensions



# **Computed Axial Tomography**

- Radon transform and sinogram
- Methods: back projection and algebraic reconstruction

# Topological Derivative

- Theoretical background
- Defects detection
- Methods: multifrequency and iterative
- Applications

## METHODOLOGY

Theory and solving techniques for inverse problems will be explained through simple examples. Then students will have to tackle more complex problems issuing from real-world applications (including from industry) by applying and modifying solving techniques. They should be able to validate their results in the context of the application.

LANGUAGE USED IN CLASS: It will depend on the audience

**IS IT COMPULSORY TO ATTEND CLASS?** Due to its interactive form, it is highly recommended to attend classes via conference system

#### BIBLIOGRAPHY

A. Kirsch, "An Introduction to the Mathematical Theory of Inverse Problems", Springer-Verlag New York [2011]

J. Mueller & S. Siltanen, "Linear and Nonlinear Inverse Problems with Practical Applications", SIAM Computational Science & Engineering (2012)

F. Natterer & F. Wübbeling, "Mathematical Methods in Image Reconstruction", Ed. SIAM (2001)

M. Bertero & P. Boccacci, "Introduction to Inverse Problems in Imaging", Ed. CRC Press (1998)

#### SKILLS

<u>Basic</u>

CG1. To have knowledge that provides a basis or opportunity for originality in developing and/or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of Industrial Mathematics.

CG3. To be able to integrate knowledge in order to state opinions using information that, although incomplete or limited, includes reflecting on social and ethical responsibilities linked to the application of their knowledge.

CG4. To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.



CG5. To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

#### <u>Specific</u>

CE3. To determine if a model of a process is mathematically well-posed and clearly formulated from a physical standpoint.

CE5. To be able to validate and interpret the results, comparing them with visualizations, experimental measurements and functional requirements of the physical or engineering system.

#### Modelling specialization

CM1. To be able to extract, using different analytical techniques, both qualitative and quantitative information from a model.

## WILL YOU BE USING A VIRTUAL PLATFORM? No

# WILL YOU BE USING ANY SPECIFIC SOFTWARE? MATLAB

**CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY:** The student will have to solve and deliver various projects on the topics developed during the course. Both the level of understanding and the acquired knowledge of the theoretical background will be evaluated (CG1, CG3), as well as the ability to delve into the topic in an autonomous way (CG5). The student will have to analyze some features of the used mathematical model (CE3) and be able to both interpret the obtained results with critical sense (CE5, CM1) and extract proper conclusions (CG4).

#### CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY: The same as for the first assessment.